B. Tech Degree I Semester Examination in Naval Architecture and Ship Building, January 2010

ST 01 S1 MATHEMATICS I

Time : 3 Hours | Maximum Marks : 100

(All questions carry equal marks)

I. (a) Find the derivative of $y = \log_e (Cos\,h\,x)$. (6)

(b) If $x + iy = Cos\,h(u + iv)$, show that $\frac{x^2}{Cos\,h^2u} + \frac{y^2}{Sin\,h^2u} = 1$. (7)

(c) Prove that $\tan h^{-1}x = \frac{1}{2}\log\,\frac{1+x}{1-x}$. (7)

OR

II. (a) Prove that $Sec(x + iy) = \frac{2\,Cos\,x\,Cos\,hy}{Cos2x + Cosh2y} + i.\frac{2\,Sin\,x\,Sin\,hy}{Cos\,2x + Cosh\,2y}$. (6)

(b) Sum the series :
$x\,Sin\,h\alpha + x^2\,Sin\,h2\alpha + x^3 Sinh3\alpha + ...\infty$. (7)

(c) Prove that hyperbolic functions are periodic and find their periods. (7)

III. (a) Show that $\tan\left(\frac{\pi}{4} + x\right) = 1 + 2x + 2x^2 + \frac{8}{3}x^3 + \frac{10}{3}x^4 + ...$ (7)

(b) If $y = x^2\,Cos\,x$, find the n[th] derivative of y. (6)

(c) If $y = \left(Sin^{-1}x\right)^2$, show that $\left(1 - x^2\right)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$. (7)

OR

IV. (a) Using Maclaurin's theorem, prove that
$\log\,(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - .... + (-1)^{n-1}\frac{x^n}{n} + ...$ (7)

(b) If $y = \tan^{-1}x$, prove that $\left(1 + x^2\right)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$. (6)

(c) Find the n[th] derivative of $e^x\,(2x+3)^3$. (7)

V. (a) Find the vertex, focus, directrix, axis and latus rectum of the parabola
$2x^2 + 5y - 3x + 4 = 0$. (7)

(b) Find the equations of the tangents to the ellipse $9x^2 + 16y^2 = 144$ from the point $(2,3)$. (7)

(c) Find the asymptotics of the hyperbola $2x^2 + 5xy + 2y^2 + 4x + 5y = 0$ and obtain its centre. (6)

OR

(Turn Over)

VI. (a) Find the radius of curvature at any point of the parabola $y^2 = 4ax$ and hence show that the radius of curvature at its vertex is equal to its semi-latus rectum. (7)

(b) Show that the locus of the poles with respect to the parabola $y^2 = 4ax$ of the tangents to the hyperbola $x^2 - y^2 = a^2$ is the ellipse $4x^2 + y^2 = 4a^2$. (7)

(c) Find the centre, eccentricity and co-ordinates of the foci of the ellipse $4x^2 + 9y^2 - 8x + 36y + 4 = 0$. (6)

VII. (a) Find the co-ordinates of the centre of curvature at any point of the parabola $y^2 = 4ax$. Hence show that its evolute is $27ay^2 = 4(x - 2a)^3$. (10)

(b) Find the envelope of the family of lines

$$y = mx + \sqrt{1 + m^2}, \ m \text{ being the parameter.}$$ (10)

OR

VIII. (a) Show that the evolute of the cycloid $x = a(\theta - Sin\theta), y = a(1 - Cos\theta)$ is another equal cycloid. (10)

(b) Find the envelope of the family of curves

$$\frac{x\,Cos\alpha}{a} + \frac{y\,Sin\alpha}{b} = 1$$ where α being the parameter and a and b constants. (10)

IX. (a) If $u = f(r)$ and $x = r\,Cos\,\theta, y = r\,Sin\,\theta$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r).$$ (10)

(b) If $z = x^3 + y^3 - 3axy$, prove that $\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$. (3)

(c) If $u = Sin^{-1}\frac{x^2 + y^2}{x + y}$, using Euler's theorem, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. (7)

OR

X. (a) If $x = e^{rCos\theta}\,Cos(r\,Sin\theta)$ and $y = e^{rCos\theta}\,Sin(r\,Sin\theta)$ prove that

$$\frac{\partial x}{\partial r} + \frac{\partial y}{\partial r} = \frac{1}{r}\left[\frac{\partial y}{\partial \theta} - \frac{\partial x}{\partial \theta}\right].$$ (7)

(b) If $u = \log\frac{x^4 + y^4}{x + y}$, show that $x\frac{\partial u}{\partial n} + y\frac{\partial u}{\partial y} = 3$. (6)

(c) The diameter and attitude of a can in the shape of a right circular cylinder are measured as 4 cms and 6 cms respectively. The possible error in each measurement is 0.1 cm. Find approximately the maximum possible error in the value computed for the volume and lateral surface. (7)

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