FIVE YEAR INTEGRATED M.SC. DEGREE II SEMESTER EXAMINATION IN PHOTONICS, SEPTEMBER 2006

CEL 1204 CLASSICAL MECHANICS

Time : 3 Hrs. Maximum marks : 50

Part A

(Answer any **five** questions)
(All questions carry **two** marks)

(5 × 2 = 10 Marks)

1. (a) What are constrained motions? Explain how the use of generalized coordinates can help to solve the constrained motion.
   (b) What is a conservative force? Check whether the force $\mathbf{F} = \mathbf{i}yz + \mathbf{j}zx + \mathbf{k}xy$ is a conservative force.
   (c) The Lagrangian for a system is $m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)/2 - mgz$. Find the equations of motion. Explain whether this Lagrangian describe any familiar physical system.
   (d) Using the ray equation find the ray trajectory in a homogeneous medium.
   (e) Define the conjugate momentà. What are cyclic coordinates?
   (f) What is a central force? Show that the angular momentum of a particle moving under the influence of a central force field with fixed center of force is conserved.
   (g) Find the conditions for a stable circular orbits for the motion under an attractive power law central potential.
   (h) Evaluate the Poisson bracket $[L_x, L_y]$ where L_x and L_y respectively are the x and y components of the angular momentum.

Part B

(Answer **all** questions)
(All questions carry **equal** marks)

(5 × 8 = 40 Marks)

2. (a) State and explain Hamilton's principle and derive Lagrange's equations of motion. (5)
   (b) Find the condition under which the total angular momentum of a system of particles is independent of the choice of the origin or the reference point. (3)

OR

3. (a) Define the center of mass of a system of particles. Obtain the equation governing the motion of the center of mass of a system of particles acted upon by external forces and internal forces obeying weak law of action and reaction. (4)
   (b) Explain the method of Calculus of variation and find the equation of the curve passing between two fixed points such that it has a minimum surface area of revolution about y-axis. (4)

(Turn over)

4. (a) Using Fermat's principle and Hamilton's principle find the expression for the optical Lagrangian. Obtain the ray equation and use it to derive law of refraction. (5)

(b) Find the Lagrangian for a simple pendulum and obtain the equation of motion. Solve the equation of motion for the case of small amplitude oscillations. (3)

OR

5. (a) If $L(q, \dot{q}, t)$ is a Lagrangian of a system with an equation of motion show that the Lagrangian

$$L(q, \dot{q}, t) + \frac{d}{dt}F(q, t)$$

also gives the same equation of motion. What is your conclusion? (4)

(b) The Lagrangian for a system is $m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)/2 + k(y\dot{x} - x\dot{y})$. Find the equations of motion. Check whether the resulting equations describe any well known dynamics. (4)

6. (a) The Lagrangian for a system is given as $m\dot{\mathbf{r}} \cdot \dot{\mathbf{r}}/2 - e\mathbf{A}(\mathbf{r}) \cdot \dot{\mathbf{r}}/c + e\phi(\mathbf{r})$. Find the corresponding Hamiltonian. What is the physical importance of this Hamiltonian? (6)

(b) Explain the conditions under which the Hamiltonian can represent the total energy of a system. (2)

OR

7. (a) With examples describe the relation between symmetry and conservation laws. Explain also the connection between symmetry and Noether's theorem. (6)

(b) Show that the Hamiltonian of a system is conserved if the Lagrangian of that system is not an explicit function of time. (2)

8. (a) With necessary details show that a three dimensional central force problem can be reduced to an equivalent one dimensional problem. (6)

(b) Show that the Kepler's second law is a consequence of the angular momentum conservation. (2)

OR

9. By considering the equivalent one dimensional problem for attractive inverse square law of force describe the classification and stability various orbits.

10. Using Lagrange's equations of motion derive Hamilton's equations motion. Comment on the difference between Hamiltonian and Lagrangian formulation of mechanics. Define the Poisson bracket $[u, v]$ and express Hamilton's equations of motion in terms of Poisson bracket notation.

OR

11. What are canonical transformations? Explain its use. Check whether the transformations $Q = \ln(1 + \sqrt{q}\cos p)$ and $P = 2(1 + \sqrt{q}\cos p)\sqrt{q}\sin p$, is canonical. If it is find the generating function $F_3(Q, p)$ for this transformation.

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