M.Sc. DEGREE (5 YEAR INTEGRATED COURSE) I SEMESTER EXAMINATION IN PHOTONICS, DECEMBER 2005

CEL 1104 MATHEMATICS I

Time: 3 Hours | Maximum marks : 50

PART – A

(Answer any ***FIVE*** questions)
(All question carry ***EQUAL*** marks)

(5 x 2 = 10)

I.

1. Find $\nabla\phi$ if $\phi = |r|$
2. Show that $\nabla^2(xy\hat{i} + yz\hat{j} + zx\hat{k}) = 0$.
3. State Cayley-Hamilton theorem for a square matrix A. Prove it for $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$.
4. Define adjoint and inverse of a square matrix.
5. Explain Cramer's rule for the solution of a system of n linear algebraic equations.
6. Write down the expression for the n^{th} root of a complex number. Find all the values of $(-1)^{1/3}$.
7. Show that $\tan 15 = 2 - \sqrt{3}$.

PART – B

(Answer ***ALL*** questions)
(All questions carry ***EQUAL*** marks)

(5 x 8 = 40)

II. A. (a) Prove that $\nabla.(\nabla \times f) = 0$ and $\nabla \times (\nabla \times f) = \nabla(\nabla.f) - \nabla^2 f$.

(b) If $\phi = x^3 + y^3 + z^3 - 3xyz$ find $\nabla.(\nabla\phi)$ and $\nabla \times (\nabla\phi)$.

OR

B. (a) Define solenoidal and irrotational vectors.

(b) Find the constants a, b, c so that
$f = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + 6y + 2z)\hat{k}$ is irrotational.

III. A. (a) State and prove Gauss's theorem of divergence.

(b) Find the work done in moving a particle in a force field $F = 3xy\hat{i} - y\hat{j} + 2zx\hat{k}$ around a curve C in the x-y plane where C is a circle of radius 2 and centre (0,0).

OR

B. (a) Evaluate $\int_c F.dr$ if $F = 3xy\hat{i} - y^2\hat{j}$ and C is the curve in the x-y plane $y = 2x^2$ from (0,0) to (1,2).

(b) Using Stokes' theorem, prove that $\int_c (yz\,dx + zxdy + xydz) = 0$ where C is the curve $x^2 + y^2 = 1$, $z = y^2$.

(Turn over)

IV. A. (a) Find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$.

(b) Solve the system of equations, $x - 2y + 3z = 2;\ 2x - 3z = 3;\ x + y + z = 6$.

OR

B. (a) Prove that the eigenvalues of a Hermitian matrix are real and those of a real skew symmetric matrix are either zero or purely imaginary.

(b) Show that $\begin{bmatrix} 1 & 1-i & 2 \\ 1+i & 3 & i \\ 2 & -i & 0 \end{bmatrix}$ is Hermitian.

V. A. (a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 2 \end{bmatrix}$.

(b) Calculate the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

OR

B. (a) Diagonalise the matrix $A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(b) If $A = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I = [0]$ where [0] is the 2×2 null matrix.

VI. A. (a) State de Moivre's theorem. Write $(1+i)$ in the in the exponential form and evaluate $(1+I)^{18}$.

(b) If $Z = 6e^{i\pi/3}$ evaluate $\left|e^{iz}\right|$.

OR

B. (a) Show that $\tan h^{-1} x = \frac{1}{2} ln \frac{1+x}{1=x}$. Express graphically the variation of *tan x* with *x*.

(b) Sum the series $S(\theta) = 1 + \cos\theta + \frac{\cos 2\theta}{2!} + \frac{\cos 3\theta}{3!} + \ldots$

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