B. Tech Degree II Semester Examination in Instrumentation, May 2007

IN 201 APPLIED MATHEMATICS - II

Time : 2 Hours

Maximum Marks : 100

PART A

(Answer ***ANY FIVE*** questions)
(All questions carry ***EQUAL*** marks)

(5 x 5 = 25)

I. (a) Find the rank of the matrix $\begin{bmatrix} 2 & 3 & 4 & -1 \\ 5 & 2 & 0 & -1 \\ -4 & 5 & 12 & -1 \end{bmatrix}$.

(b) If A is an orthogonal matrix, prove that $|A| = \neq 1.|$

(c) Determine grad ϕ where ϕ is given by $\phi = 3x^2y - y^3z^2$ at the point $(1,-2,-1)$.

(d) Find the Laplace Transform of $e^{-3t}\left(Cos\,4t + 3\,Sin\,4t\right)$.

(e) Discuss the convergence of the series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots\ldots + \frac{1}{n(n+1)} + \ldots\ldots$ to $\propto$

(f) State D'Alembert's Ratio Test and using this test, check whether the series $\sum \frac{n^1}{n^n}$ is convergent.

(g) Solve $\frac{dy}{dx} = \frac{x(2\log x + 1)}{S\,\text{in}\,y + y\,Cos\,y}$.

PART B

(Answer ***ALL*** questions)
(All questions carry ***EQUAL*** marks)

(5 x 15 = 75)

II. (a) Test the consistency of the equations $x - y + z = 2;\ 3x - y + 2z = -6;\ 3x + y + z = -18$ and solve if consistent .

(b) Using Cayley-Hamilton theorem find the inverse of the matrix

$$\begin{pmatrix} -1 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}.$$

OR

III. (a) Find the eigen values and eigen vectors of the matrix

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

(b) Prove that the matrix $A = \begin{pmatrix} 0 & -2 & -2 \\ -1 & 1 & 2 \\ -1 & -1 & 2 \end{pmatrix}$ is not diognalisable .

(Turn Over)

IV. (a) Find the value of m if $a=(x+2y)i+(my+4z)j+(5z+6x)k$ is solenoidal.

(b) Evaluate by using Stoke's theorem.

$$\int_c (yzdx+zxdy+xydz) \text{ where C is the curve } x^2+y^2=1, z=y^2.$$

OR

V. (a) If $\phi=x^3+y^3+z^3-3xyz$, find

(i) div $(\text{grad}\,\phi)$ (ii) curl $(\text{grad}\,\phi)$.

(b) Use Gauss divergence theorem to evaluate $\iint_S f.nds$ where $f=yi+xj+z^2k$ for the cylindrical region S given by $x^2+y^2=a^2$; $z=0$ and $z=h$.

VI. (a) Find the Laplace transform of

(i) $Sin\ 2t\ Cos\ 3t$ (ii) t^3e^{-3t}

(b) Find the inverse Laplace transform of

(i) $\dfrac{2s^2-4}{(s+1)(s-2)(s-3)}$

(ii) $\dfrac{1}{s(s+1)^3}$

OR

VII. (a) Solve the equation :

$y''-3y'+2y=4t+e^{3t}$, where $y(0)=1$ and $y'(0)=-1$.

(b) A voltage Ee^{-at} is applied at $t=0$ a circuit of inductance L and resistance R. Show that the current at time t $\dfrac{E}{R-aL}\left(e^{-at}-e^{-Rt/L}\right)$.

VIII. Examine the conveyance of the series

(i) $\sum \dfrac{\sqrt{(n+1)}-\sqrt{n}}{n^p}$ (ii) $\sum\left(\dfrac{n}{n+1}\right)^{n^2}$

IX. (a) Find the Maclaurin's series representation of $\sin z$

(b) Expand $f(z)=\dfrac{1}{1-z}$ in a Taylor series with ……

$z_0=2i$.

X. (i) Solve the differential equation

$$\frac{dy}{dx}=Cot(y+x)-1$$

(ii) Show that the differential equation $\dfrac{dy}{dx}=\dfrac{\tan y-y-2xy}{\sec^2 y-x\tan^2 y+x^2+2}$ in an exact equation.

OR

XI. Solve

(i) $(D^2-13D+12)y=e^{-2x}$

(ii) $\dfrac{d^2y}{dx^2}+4\dfrac{dy}{dx}+4y=e^{3x}+Cos\,5x$.

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