MCA DEGREE I SEMESTER EXAMINATION, NOVEMBER 2008

PROBABILITY AND STATISTICS (SUPPLEMENTARY)

Time: 3 Hours | Maximum marks : 50

PART A

(Answer ***ALL*** questions)
(All questions carry ***TWO*** marks)

(15 x 2 = 30)

I. (a) Define Sample space and Events. Give examples.

(b) For any two events A and B, prove that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(b) Let A and B be two independent events defined on some probability space, and let $P(A) = 1/3$, $P(B) = 3/4$.

Find (i) $P(A/B)$ (ii) $P(A/A \cup B)$

II. (a) Let X be the number of heads in three tosses of a coin. Describe the sample space Ω. What are the values that X assigns to point of Ω? What are the events $\{X \le 2.75\}$ and $\{0.5 \le X \le 1.72\}$?

(b) Define Binomial distribution. If $n = 10$, $p = 1/3$, obtain the mean and variance of the binomial distribution $B(n, p)$.

(c) Verify whether the following functions represent distribution functions

(i) $F(x) = 1 - e^{-x}\ x > 0.$

(ii) $F(x) = \frac{1}{\pi}\tan^{-1} x,\ -\infty < x < \infty.$

III. (a) Find the moment generating function and hence the mean and variance of random variable X with p.m.f., $P(X = k) = \begin{cases} \frac{e^{-\lambda}\lambda^k}{k!} & k = 0,1,2,\ldots\ldots \\ 0 & \text{otherwise} \end{cases}$

(b) Define characteristic function. Check whether $\phi(t) = (1+t^2)^{-1}, -\infty < t < \infty$ is a characteristic function.

(c) Explain the weak law of large numbers.

IV. (a) What are the different measures of dispersion? Comment on the uses of them.

(b) Obtain the m.g.f. of binomial distribution.

(c) Distinguish between linear and non-linear regression.

V. (a) Define Simple Random Sampling. What is its importance?

(b) Write a short note on systematic sampling.

(c) Define the terms :

(i) Rejection region (ii) Type I error
(iii) Type II error (iv) Power of a Test.

(Turn Over)

PART B

(All questions carry ***FOUR*** marks)

(5 x 4 = 20)

VI. A. State and prove Bayes theorem.

OR

B. If A,B,C are any three events, prove that $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$.

VII. A. Let X follows a Poisson distribution with parameter λ . Obtain the distribution of the random variable $Y = X^2 + 3$.

OR

B. If $X \sim N(15,16)$, find

(i) $P(X \leq 12)$ (ii) $P(10 \leq X \leq 17)$

(iii) $P(10 \leq X \leq 19 / X \leq 17)$ (iv) $P(|X - 15| > 5)$.

VIII. A. State and prove the 'central limit theorem 'for independent identically distributed random variables.

OR

B. Distinguish between strong law of large numbers (SLLN) and weak laws of large numbers (WLLN). Verify whether the laws of large numbers hold for the following : $\{X_k\}$ with $P\left(X_k = \pm 2^k\right) = \frac{1}{2}$.

IX. A. Define efficiency of estimators. If $X \sim N\left(\mu, \sigma^2\right)$, show that the sample mean $\overline{X}$ and sample median M_d are unbiased estimators of μ . Which estimator is more efficient? Justify your answer.

OR

B. Calculate the coefficient of correlation and obtain the lines of regression for the following data. :

x	1	2	3	4	5	6	7	8	9
Y	9	8	10	12	11	13	14	16	15

Also obtain the estimate of y when $x = 6.2$.

X. A. State and prove Neyman-Pearson fundamental lemma on testing of hypothesis.

OR

B. Samples of final examination scores for the two statistics classes with different instructors showed the following results :

Instructor A's Class	Instructor B's Class
$n_1 = 12$	$n_2 = 15$
$\overline{x}_1 = 72$	$\overline{x}_2 = 78$
$s_1 = 8$	$s_2 = 10$

With $\alpha = 0.05$, use the p-value to test whether or not these data are sufficient to conclude that the mean grades differ for the two classes.