MCA DEGREE I SEMESTER EXAMINATION, DECEMBER 2006

CAS 2102 PROBABILITY AND STATISTICS

Time: 3 Hours　　　　Maximum marks : 50

PART A

(Answer ***ALL*** questions)
(Each question carries ***TWO*** marks)

(15 x 2 = 30)

I. (a) If A_1 and A_2 are independent events, show that A_1^c and A_2^c are independent.

(b) A, B and C shoot a target. The probabilities of hitting the target are p_1, p_2 and p_3 respectively. What is the probability that at least one of them hits the target ?

(c) A rental car service facility has 10 foreign cars and 15 domestic cars on a particular day. Because there are so few mechanics working on that day, only 6 cars be serviced. If the 6 are chosen at random, what is the probability that at most 3 foreign cars are selected.

II. (a) Define binomial distribution. Obtain its mean and variance.

(b) A college professor never finishes his lecture before the bell rings to end the period and always finishes his within 2 minutes after the bell rings. Let X=the time that elapses between the bell and the end of the lecture and suppose that the probability density function of X is

$$f(x)=\begin{cases} k\ x^2 & 0 \le x \le 2 \\ 0 & Otherwise \end{cases}$$

(i) Find the value of k

(ii) What is the probability that the lecture ends within 1 minute of the bell ringing ?

(c) Define exponential distribution. Show that its mean is equal to the standard deviation.

III. (a) Obtain characteristic function of Poisson distribution and hence find its mean.

(b) For normal distribution, show that the measure of sknewness is zero.

(c) State and prove weak law of large numbers.

IV. (a) Define (i) mean (ii) median and (iii) mode. Compare these measures as measures of central tendency.

(b) What are the measures of dispersion ? Show the measure 'variance' is independent of the location.

(c) Define simple linear regression model. Explain the method for estimating parameters of the model.

V. (a) Distinguish between sampling and census.

(b) Define (i) type I error, (ii) type II error and (iii) critical region.

(c) Explain, briefly, a method for testing mean of a normal distribution when variance is known.

PART B

(Answer ***ALL*** questions)
(Each question carries ***FOUR*** marks)

(5 x 4 = 20)

VI. A. If A, B and C are three events, show that $P(A\cup B\cup C)\le P(A)+P(B)+P(C)$.

OR

B. State and prove Bayes theorem.

(Turn over)

VII. A. Prove or disprove: For Poisson distribution, mean is equal to the variance of the distribution.

OR

B. Define normal distribution. In a normal distribution 40% of the values are below 50 and 20% are above 60. find the mean and variance of the distribution.

VIII. A. Define characteristic function. State its important properties. Obtain characteristic function of Cauchy distribution with $f(x) = \dfrac{1}{\Pi\left(1+X^2\right)}$ $\quad -\infty < X < \infty$.

OR

B. State and prove central limit theorem.

IX. A. Define moment generating function of a random variable X. prove or disprove: Moment generating function always exists.

OR

B. What are regression equations? From the data given below, obtain the regression equation of X on Y.

X:	5	10	15	20	25	30	45	60
Y:	16.3	9.7	8.1	4.2	3.4	2.9	1.9	1.3

X. A. Define (i) simple random sampling (ii) stratified random sampling and (iii) systematic sampling. If $\overline{y}$ is the estimate of the population mean show that $\underset{Strs}{V}(\overline{y}) \le \underset{Srs}{V}(\overline{y})$; where 'strs' denote the stratified sampling and 'srs' denote the simple random sampling.

OR

B. Let μ_1 and μ_2 be true average densities for two different types of brick. Assuming normality of the two density functions, test $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$ using the data, $n_1 = 6$, $\overline{x}_1 = 22.73$ $S_1 = 0.164$, $n_2 = 5$, $\overline{x}_2 = 21.95$ and $S_2 = 0.24$.

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