M.C.A. DEGREE I SEMESTER EXAMINATION, DECEMBER 2006

CAS 2105 DISCRETE MATHEMATICAL STRUCTURES

Time: 3 Hours | Maximum marks : 50

PART – A

*(Answer **ALL** questions)*
*(Each question carries **TWO** marks)*

I. (a) Let l be any collection of sets. Is the relation of set inclusion $\subseteq$ a partial order on l ?

(b) Give an example of an infinite lattice L with finite length.

(c) Write the dual of each Boolean equation: (i) $(a * 1) * (0 + a') = 0$; (ii) $a + a'b = a + b$.

II (a) Let p be "It is cold" and let q be "It is raining". Give a simple verbal sentence which describes each of the following statements: (i) $\neg p$; (ii) $p \wedge q$; (iii) $p \vee q$; (iv) $q \vee \neg p$.

(b) Determine the contra positive of each statement:

(i) If John is a poet, then he is poor;

(ii) Only if Mare studies will he pass the test

(c) Let $p(x)$ denote the sentence "$x + 2 > 5$ ". State whether or not $p(x)$ is a propositional function on each of the following sets: (i) **N**, the set of positive integers; (ii) $M = \{-1, -2, -3, \ldots\}$.

III. (a) Let A be {a,b}. Construct an automaton M such that L (M) will consist of those w which begin with a and end in b.

(b) Prove that any language L over a countable alphabet is countable

(c) Explain the notion of a finite state automaton with an example.

IV. (a) What is a regular expression and a language determined by a regular expression?

(b) State pumping Lemma. Give one of its applications.

(c) Show that $L = \{\omega.\omega / \omega \in (0,1)^*\}$ is non regular.

V. (a) Find the language $L(G)$ generated by the grammar G with variables S, A, B, terminals a,b, and productions $S \rightarrow aB$, $B \rightarrow b$, $B \rightarrow bA$, $A \rightarrow aB$.

(b) Classify Chomsky criteria for grammars.

(c Does a derivation tree exist for any word w derived from the start symbol S in a grammar G ? explain.

(Turn over)

PART – B

*(Answer **ALL** questions)*
*(Each question carries **FOUR** marks)*

VI. A. Let A be a set of nonzero integers and let $\approx$ be the relation on $A \times A$ defined by

$(a,b) \approx (c,d)$ whenever $ad = bc$.

Prove that $\approx$ is an equivalence relation.

OR

B. Express the Boolean expression $E(x,y,z)$ as a sum-of- products and then in its complete sum-of-products form: (a) $E = x(xy' + x'y + y'z)$; (b) $E = z(x' + y) + y'$

VII A. Consider the conditional proposition $p \rightarrow q$. The simple propositions $q \rightarrow p$, $\neg p \rightarrow \neg q$, and $\neg q \rightarrow \neg p$ are called, respectively, the *converse, inverse,* and *contra positive* of the conditional $p \rightarrow q$. Which if any of these propositions are logically equivalent to $p \rightarrow q$?

OR

B. Prove that the following argument is valid; $p \rightarrow \neg q,\ r \rightarrow q,\ r \vdash \neg p$.

VIII A. Describe the fundamental relation between the regular language & automation. Illustrate this with an example.

OR

B. Construct the next stage functions so that the following defines an automaton with two input symbols and three states:

(i) $A = \{a,b\}$, input symbols
(ii) $S = \{so, s_1, s_2\}$, internal states.
(iii) $Y = \{s_0, s_1\}$, "yes" states.
(iv) s_0, initial state.

IX A. Explain a regular language

Let L be the language on $A = \{a,b\}$ defined by $L = \{b, a^r b, ba^s, a^r ba^s : r > 0, s > 0$. Find a regular expression r such that $L = L(r)$.

OR

B. Show that the language $L = \{a^m b^m$: m positive$\}$ is not regular.

X A. Describe different types of grammars. Determine the type of grammar for the following productions:

(i) $S \rightarrow aB,\ B \rightarrow bA,\ B \rightarrow b,\ B \rightarrow a,\ A \rightarrow aB,\ A \rightarrow a$.
(ii) $S \rightarrow aAB,\ AB \rightarrow a,\ A \rightarrow b,\ B \rightarrow AB$.

OR

B. Show that any context free language without ε is generated by a grammar in which all the productions are of the form $A \rightarrow BC$ or $A \rightarrow a$. Here A, B and C are variables and **a** is a terminal

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