M.C.A. DEGREE I SEMESTER EXAMINATION, DECEMBER 2005

CAS 2105 DISCRETE MATHEMATICAL STRUCTURES

Time: 3 Hours Maximum marks : 50

PART - A

(Answer ***ALL*** questions)
(Each question carries ***TWO*** marks)

I. (a) Let $A = \{7,8,9\}$. Determine all the partitions of the set A.

(b) Show that a Lattice with three or four elements is a chain.

(c) Define a Boolean algebra and show that $(P(A), \cup \cap \subseteq)$ is a Boolean algebra.

II. (a) Find the truth value of the following propositions if p and r are true and q is false
(i) $(\sim p \vee q) \wedge r$ (ii) $\sim (p \vee q) \wedge r$

(b) If $p \Rightarrow q$ is false, can you determine the truth value of $(\sim p) \vee (p \Leftrightarrow q)$? Explain your answer.

(c) Prove or disprove: The sum of any five consecutive integers is divisible by five.

III. (a) Define and give examples of the terms
(i) Finite automation (ii) transition diagram

(b) Find regular expression of the Language containing all strings of O's and I's that have even length.

(c) Draw the transition diagram for the NFA given below

States \ Inputs	0	1
q_0	$\{q_0, q_3\}$	$\{q_0, q_1\}$
q_1	ϕ	$\{q_2\}$
q_2	$\{q_2\}$	$\{q_2\}$
q_3	$\{q_4\}$	ϕ
q_4	$\{q_4\}$	$\{q_4\}$

IV. (a) State the pumping lemma for regular sets.

(b) Show that $L = \{\omega.\omega / \omega \in (0,1)^*\}$ is non regular.

(c) Let $M = (Q, \Sigma, \delta, q_0, A)$ where $Q = (q_0, q_1, q_2, q_3)$, $\Sigma = \{a,b\}$ $A = \{q_3\}$ δ is given by the following table

	a	b
q_0	$\{q_0, q_1\}$	$\{q_0,\}$
q_1	$\{q_2\}$	$\{q_2\}$
q_2	$\{q_3\}$	$\{q_3\}$
q_3	ϕ	ϕ

Find the language corresponding to the finite qutomaton M.

V. (a) Define context free grammar. Give an example.

(b) If G is a grammar $S \rightarrow SbS/a$, verify whether G is ambiguous or not.

(c) Classify Chomsky criteria for grammars.

(Turn over)

PART - B

(Answer ***ALL*** questions)
(Each question carries ***FOUR*** marks)

VI. A. In a Lattice $(L, \leq)$ prove that for $a, b, c \in L$

(i) $(a * b) \oplus (a * c) \leq a * (b \oplus (a * c))$

(ii) $(a \oplus b) * (a \oplus c) \geq a \oplus (b * (a \oplus c))$

OR

B. Prove that a mapping from one Boolean algebra to another that preserves the operation $\oplus$ also preserves the operation $*$.

VII. A. Compute the truth table of the statement
$(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$

OR

B. Prove by mathematical induction that if a set *A* has *n* elements, then *P*(*A*) has 2^n elements.

VIII. A. Show that if *L* is a language accepted by a *NFA*, then there is a *DFA* that accepts *L*.

OR

B. Construct *DFA* equivalent to the *NFA* $(\{p, q, r, s\}, \{0, 1\}, 6, p, \{s\})$ where δ is given by

	0	1
p	p,q	p
q	r	r
r	s	-
s	s	s

IX. A. State and prove Myhill-Nerode theorem.

OR

B. Show that there is an algorithm to determine if two automata are equivalent.

X. A. Show that any context free language without ε is generated by a grammar in which all the productions are of the form $A \rightarrow BC$ or $A \rightarrow a$. Here *A*, *B* and *C* are variables and a is a terminal.

OR

B. What set of strings are defined by the following grammar:

Terminal symbols	:	λ, 0 and 1
Non terminal symbols	:	*S* and *E*
Starting symbol	:	*S*
Production rules	:	$S \Rightarrow 0S0$, $S \Rightarrow 1S1$, $S \Rightarrow E$
		$E \Rightarrow \lambda$, $S \Rightarrow 0$, $E \Rightarrow 1$

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