M.C.A. DEGREE I SEMESTER EXAMINATION, DECEMBER 2005

CAS 2101 COMBINATORICS AND GRAPH THEORY

Time: 3 Hours | Maximum marks : 50

PART – A

(Answer ***ALL*** questions)
(Each question carries ***TWO*** marks)

I. (a) Find the number of ways of seating n women and n men at a round table so that between every two women there is a man.
(b) Give a combinatorial proof of Pascal's identity
C(n,r) = C(n-1,r) + C(n-1,r-1)
(c) Find the rook polynomial of a 2 x 2 board by use of the expansion formula.

II. (a) What are the properties of a symmetric block design?
(b) Define Hadamard matrix.
(c) Define a Steiner triple system.

III. (a) Define isomorphic graphs. Give example.
(b) What is a Bipartite Graph? Is a tree a bipartite graph?
(c) Give two characteristic properties of a tree.

IV. (a) Define planar graph. Write the formula for finding the number of regions in any planar graph.
(b) Define the adjancy matrix of a Graph.
(c) What do you mean by a chromatic polynomial of a graph? Write the chromatic polynomial of a tree.

V. (a) Define a directed Graph.
(b) State Cayley's Theorem
(c) Define Arborescence.

PART – B

(Answer ***ALL*** questions)
(Each question carries ***FOUR*** marks)

VI. A. Solve $a_n = 3\, a_{n-1} + 2$ with $a_0=1$

OR

B. State Hall's marriage theorem. Why is it so called? Illustrate with an example.

VII. A. What are block designs? Explain by mentioning examples.

OR

B. Define Lattice. Show that Leech's lattice is actually a lattice.

VIII. A. State and prove max-flow men-cut theorem.

OR

B. (i) Define a Hamiltonian Graph.
(ii) Give a sufficient condition for any Graph to have a Hamiltonian circuit,
(iii) Give an example of a graph which is Hamiltonian but not Eulesian

(Turn over)

IX. A. Prove that every tree with 2 or more vertices is 2 chromatic.

OR

B. (i) Prove that in any simple connected graph with a f regions, n vertices and e edges $e \le 3n - 6$

(ii) check whether K_5 is planar.

X. A. Prove that every complete tournament has a directed Hamiltonian Path.

OR

B. (i) State polya's counting theorem

(ii) Use the above theorem to find how many ways can 4 identical balls be arranged in the corner's of a cube.

[Two arrangements are considered same if by any rotation of the cube they can be transformed into cash other].

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