ST.4/08.3(a)

M.Sc. DEGREE IV SEMESTER EXAMINATION IN STATISTICS, APRIL 2008

STA 2404 OPERATIONS RESEARCH

Time: 3 Hours | Maximum marks : 50

PART-A

(Answer **any five** questions)

(All questions carry **equal** marks)

(5x2=10)

I

(1) Prove that the set of constraints of a linear programming problem (L.P.P) forms a convex set.

(2) Distinguish between a solution and a feasible solution to a L.P.P.

(3) What is degeneracy in a L.P.P.?

(4) Prove that if X_1 and X_2 are two alternate optimal solutions to a L.P.P, then $X= \lambda X_1 +(1-\lambda)X_2$ is also an optimal solution.

(5) State the transportation problem.

(6) What is an unbalanced transportation problem? How do you solve it?

(7) State the necessary and sufficient conditions for a point x^o to be a minimum point of $f(x)$, $x \in R^n$.

(8) Stating the assumptions derive the EOQ formula.

(9) Explain the terms 'lead time' and 'buffer stock'.

(10) Explain the characteristics of the $M|E_k|1$ queueing system.

PART-B

(Answer **ALL** questions)

(All questions carry **equal** marks)

(4x10=40)

II. A. (a) Derive a set of sufficient conditions under which a basic feasible solution becomes optimal.

(b) What are artificial variables? Explain the Big-M method of solving a L.P.P.

OR

(Turn over)

B(a) Solve the following L.P.P. using two-phase method

$$\text{Minimize } Z = 3x_1 - 4x_2$$

$$\text{Subject to} \quad 3x_1 + 11x_2 \geq 20$$
$$4x_1 - 5x_2 \leq 12$$
$$5x_1 + 9x_2 \leq 30$$

$$x_1, x_2, \geq 0$$

(b) Describe the revised simplex technique of solving a L.P.P. State its advantages over simplex technique.

III.A. (a) Explain the concept of duality in L.P.P. Write down the dual of the L.P.P. given in Q. II B(a).

(b) Explain the Vogel's method of finding an initial feasible solution to a transportation problem. Explain the uv-method of improving it.

OR

B (a) Prove the following:

(i) The dual of the dual is primal L.P.P.

(ii) If the primal has unbounded solution then the dual will have no feasible solution.

(b) Show that every minor of the transportation matrix A has the value either 0 or ± 1.

IV. A (a) Derive the Kuhn-Tucker necessary conditions for solving the following nonlinear programming problem.

Minimize $f(x), x \in R^n$.

Subject to $g_i(x) \leq b_i$, $i=1,2,\ldots m$

(b) Solve the following nonlinear programming problem using Lagrangian method.

$$\text{Minimize } f(x) = 6x_1^2 + 8x_2^2 + 2x_1x_2 - 2x_1 - x_2$$

$$\text{Subject to} \quad 5x_1 + 6x_2 = 12.$$

(Contd.....3)

OR

B (a) Describe Wolfe's modified simplex method of solving a quadratic programming problem.

(b) Obtain the Kuhn-Tucker necessary conditions for the following NLP:

$$\text{Minimize } f(x) = 4x_1^2 + 5x_2^2 + 10x_1x_2 - 6x_1 - x_2$$

$$\text{Subject to} \quad 3x_1 + 11x_2 \le 16$$
$$4x_1 - 5x_2 \le 5$$

V.A (a) Derive the difference-differential equations for the M|M|s queue and obtain the queue length distribution in steady state conditions.

(b) Derive the formula for the optimal order quantity for a single period deterministic inventory model assuming that the demand and replenishment occur at a uniform rate.

OR

B (a) Making the imbedded Markov chain analysis derive the Pollaczek-Khintchine formula for the M|G|1 queueing system.

(b) In a single period stochastic inventory problem given the initial stock K = Rs. 600, production cost c= Rs. 4 holding cost is h= Rs.0.5 and penalty cost is Rs. 6 and the probability distribution of demand is given by

$$\varphi_D(\xi) = 1/100, \quad 0 < \xi < 100$$

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