BT SH-I-11.18-1837 Reg. No. [ ]

C

B.Tech. Degree I Semester Examination in Naval Architecture and Ship Building November 2018

ST 18 1102 MATHEMATICS I
(2018 Scheme)

Time: 3 Hours Maximum Marks: 100

PART A
(Answer *ALL* questions)

(5 × 4 = 20)

I. (a) Prove that $(\cosh x + \sinh x)^n = \cosh nx + \sinh nx$.

(b) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $xy + yz + zx = 1$.

(c) Find the n[th] derivative of $x^2 \sin x$.

(d) Identify the conic $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and find its eccentricity and latus rectum.

(e) Expand $2x^3 + 7x^2 + x - 1$ in powers of $(x-2)$ by Taylor series method.

PART B

(5 × 16 = 80)

II. (a) If $x + iy = \tan(A + iB)$ prove that $x^2 + y^2 + 2x\cot 2A = 1$. (8)

(b) Prove that $\sinh 3x = 3\sinh x + 4\sinh^3 x$. (8)

OR

III. (a) Prove that $\cosh^{-1}(x) = \log_e\left(x + \sqrt{x^2 - 1}\right)$. (8)

(b) If $\tan\left(x/2\right) = \tanh\left(x/2\right)$, then prove that $\cos x \cosh x = 1$. (8)

IV. (a) Expand $e^x \sin x$ up to term containing x^4. (7)

(b) If $y = a\cos(\log x) + b\sin(\log x)$, then show that (9)

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + \left(n^2 + 1\right)y_n = 0.$$

OR

V. (a) Expand $\sin x$ in power of $\left(x - \pi/2\right)$. (6)

(b) If $\cos^{-1}\left(y/b\right) = \log\left(x/n\right)^n$, then prove that (10)

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0.$$

(P.T.O.)

VI. (a) Prove that the locus of the point of intersection of two numbers to the parabola $y^2 = 4ax$ which are perpendicular to each other is the curve $y^2 = a(x-3a)$. (8)

(b) Identify the conic $9x^2 - 16y^2 + 72x - 32y - 16 = 0$ and find its centre, eccentricity, foci and directrix. (8)

OR

VII. (a) Find the condition that the straight line $lx + my + n = 0$ may be a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (8)

(b) Show that the locus of the poles with respect to the parabola $y^2 = 4ax$ of the tangents of the hyperbola $x^2 - y^2 = a^2$ is the ellipse $4x^2 + y^2 = 4a^2$. (8)

VIII. (a) Find the envelope of the family of curves $y = mx + am^p$ where m is a parameter. (7)

(b) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (9)

OR

IX. (a) Find the envelope of a system of concentric and coaxial ellipses of constant area. (7)

(b) Prove that the evolute of the hyperbola $2xy = a^2$ is $(x+y)^{2/3} - (x-y)^{2/3} = 2a^{2/3}$. (9)

X. (a) If $z = e^{ax+by} f(ax - by)$ prove that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$. (10)

(b) If $x^y + y^x = c$, find $\frac{dy}{dx}$. (6)

OR

XI. (a) State Euler's theorem on homogenous function and verify it for $u = \frac{x^2\left(x^2 - y^2\right)^3}{\left(x^2 + y^2\right)^3}$. (8)

(b) If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$ where $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ is the Jacobian of u, v, w with respect to x, y, z. (8)

***